

Homework 4 Solutions

Oppenheim Book

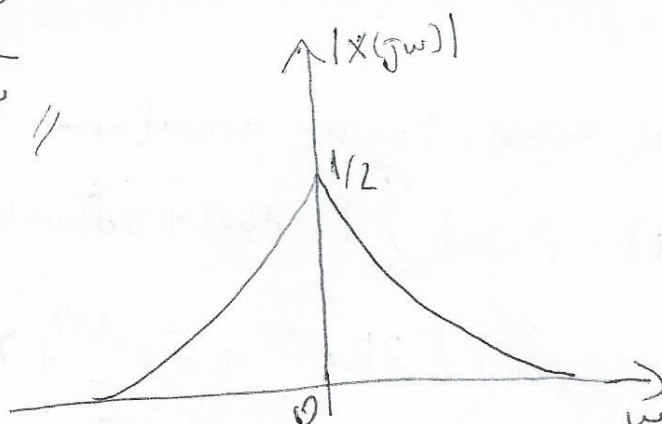
4.1. a-) Let $x(t) = e^{-2(t-1)} u(t-1)$. Then the Fourier transform

$X(j\omega)$ of $x(t)$ is:

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-2(t-1)} u(t-1) e^{-j\omega t} dt$$

$$= \int_1^{\infty} e^{-2(t-1)} e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega}}{2+j\omega}$$



b-) Let $x(t) = e^{-2|t-1|}$. Then the Fourier transform $X(j\omega)$ of $x(t)$ is:

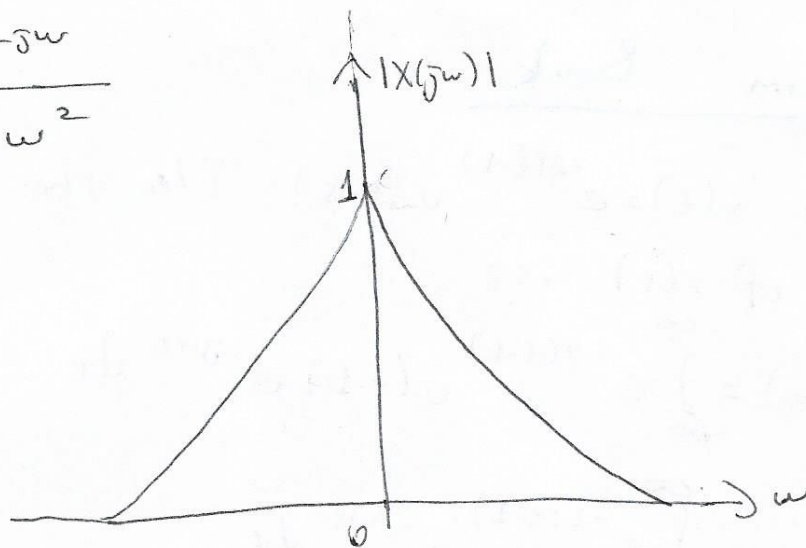
$$X(j\omega) = \int_{-\infty}^{\infty} e^{-2|t-1|} e^{-j\omega t} dt$$

$$= \int_1^{\infty} e^{-2(t-1)} e^{-j\omega t} dt + \int_{-\infty}^1 e^{2(t-1)} e^{-j\omega t} dt$$

$$= e^{-j\omega} / (2 + j\omega) + e^{j\omega} / (2 - j\omega)$$

(2)

$$= \frac{4e^{-j\omega}}{4 + \omega^2}$$



4.4-7 a-) The inverse Fourier transform is

$$x_1(t) = (1/2\pi) \int_{-\infty}^{\infty} [2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)] e^{j\omega t} d\omega$$

$$= (1/2\pi) [2\pi e^{j0t} + \pi e^{j4\pi t} + \pi e^{-j4\pi t}]$$

$$= 1 + (1/2)e^{j4\pi t} + (1/2)e^{-j4\pi t} = 1 + \cos(4\pi t)$$

b-) The inverse Fourier transform is

$$x_2(t) = (1/2\pi) \int_{-\infty}^{\infty} X_2(j\omega) e^{j\omega t} d\omega$$

$$= (1/2\pi) \int_0^2 2e^{j\omega t} d\omega + (1/2\pi) \int_{-2}^0 (-2)e^{j\omega t} d\omega$$

$$= (e^{j2t} - 1) / (\pi j t) - (1 - e^{-j2t}) / (\pi j t)$$

$$= \frac{-4j \sinh^2 t}{\pi t}$$

4.5. From given information,

(3)

$$\begin{aligned}
 x(t) &= (1/2\pi) \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \\
 &= (1/2\pi) \int_{-\infty}^{\infty} |X(j\omega)| e^{j\angle\{X(j\omega)\}} e^{j\omega t} d\omega \\
 &= (1/2\pi) \int_{-3}^3 2 e^{-j\omega/2 + j\pi} e^{j\omega t} d\omega \\
 &= \frac{-2}{\pi(t-3/2)} \sin[3(t-3/2)]
 \end{aligned}$$

The signal $x(t)$ is zero when $3(t-3/2)$ is a nonzero integer of π . This gives

$$t = \frac{k\pi}{2} + \frac{3}{2} \quad \text{for } k \in \mathbb{I}, \text{ and } k \neq 0.$$

4.19. We know that

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Since it is given that $y(t) = e^{-3t} u(t) - e^{-4t} u(t)$, we can compute $Y(j\omega)$ to be

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

Since, $H(j\omega) = \frac{1}{3+j\omega}$, we have

$$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4 + j\omega}$$

(4)

Taking the inverse Fourier transform of $X(j\omega)$, we have
 $x(t) = e^{-4t} u(t)$

4.21. a-) The given signal is

$$e^{-\alpha t} \cos(\omega_0 t) u(t) = \frac{1}{2} e^{-\alpha t} e^{j\omega_0 t} + \frac{1}{2} e^{-\alpha t} e^{-j\omega_0 t} u(t)$$

Therefore,

$$X(j\omega) = \frac{1}{2(\alpha - j\omega_0 + j\omega)} + \frac{1}{2(\alpha + j\omega_0 + j\omega)}$$

b-) The given signal is

$$x(t) = e^{-3t} \sin(2t) u(t) + e^{3t} \sin(2t) u(-t)$$

We have

$$x_1(t) = e^{-3t} \sin(2t) u(t) \xleftrightarrow{\text{FT}} X_1(j\omega) = \frac{1/2j}{3 - j^2 + j\omega} - \frac{1/2j}{3 + j^2 + j\omega}$$

Also,

$$x_2(t) = e^{3t} \sin(2t) u(-t) = -x_1(t) \xleftrightarrow{\text{FT}}$$

$$X_2(j\omega) = -X_1(-j\omega) = \frac{1/2j}{3 - j^2 - j\omega} - \frac{1/2j}{3 + j^2 - j\omega}$$

Therefore,

$$X(j\omega) = X_1(j\omega) + X_2(j\omega) = \frac{3j}{9 + (\omega + 2)^2} - \frac{3j}{9 + (\omega - 2)^2}$$

4.21 - h) on Page 6

$$4.22 - b-) \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (X(j\omega)) e^{j\omega t} d\omega$$

$$x(t) = \frac{1}{2} e^{-j\pi/3} f(t-4) + \frac{1}{2} e^{j\pi/3} f(t+4).$$

4.26. -a-)

(i) We have

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) = \left[\frac{1}{(2+j\omega)^2} \right] \left[\frac{1}{4+j\omega} \right]$$

$$= \frac{1/4}{4+j\omega} - \frac{(1/4)}{2+j\omega} + \frac{(1/2)}{(2+j\omega)^2}$$

Taking the inverse Fourier transform we obtain

$$y(t) = \frac{1}{4} e^{-4t} u(t) - \frac{1}{4} e^{-2t} u(t) + \frac{1}{2} t e^{-2t} u(t)$$

(iii)

We have

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) = \left[\frac{1}{1+j\omega} \right] \left[\frac{1}{1-j\omega} \right]$$

$$= \frac{1/2}{1+j\omega} + \frac{1/2}{1-j\omega}$$

Taking the inverse Fourier transform, we obtain (6)

$$y(t) = \frac{1}{2} e^{-|t|}$$

7.2. From the Nyquist theorem, we know that the sampling frequency in this case must be at least $\omega_s = 2000\pi$. In other words, the sampling period should be at most $T = \frac{2\pi}{\omega_s} = 1 \times 10^{-3}$. Clearly, only (a) and (c) satisfy this condition.

7.3. -a-) We can easily show that $X(j\omega) = 0$ for $|\omega| > 4000\pi$. Therefore, the Nyquist rate for this signal is:
 $\omega_N = 2(4000\pi) = 8000\pi$.

4.21 n) if $x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$,

then $x(t) = 2x_1(t) + x_1(t-1)$.

Therefore $X(j\omega) = X_1(j\omega) (2 + e^{-j\omega})$
 $= \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi) (2 + (-1)^k)$

Soliman Book

$$4.1. a-) \quad x(-t) \xrightarrow{FT} X(-\omega)$$

$$b-) \quad x_e(t) = \frac{x(t) + x(-t)}{2} \xrightarrow{FT} \frac{X(\omega) + X(-\omega)}{2}$$

4.2. a-) If $x(t)$ is absolutely integrable, then it has Fourier transform.

Using eqn. 4.2.8

$$\begin{aligned} \int_{-\infty}^{\infty} |x(t)| dt &< \infty \\ &= \int_{-\infty}^{\infty} |e^{-2t}| \cdot u(-t) dt \\ &= \int_{-\infty}^0 |e^{-2t}| dt = \left. -\frac{1}{2} e^{-2t} \right|_{-\infty}^0 = \infty \end{aligned}$$

So, $x(t)$ is not absolutely integrable and it doesn't have Fourier Transform.

b-) By following some procedure in previous solution, $x(t)$ is not absolutely integrable. No Fourier Transform.

4.2-c) $x(t) = \cos\left(\frac{\pi}{t}\right)$ is not well-behaved at $t=0$, no $X(\omega)$

4.5. a-) $\mathcal{F}\{x(-t)\} = X(-\omega)$ (8)

$$X(-\omega) = \text{rect}\left[\frac{(-\omega-1)/2}{1}\right] = \text{rect}\left[\frac{(\omega+1)/2}{1}\right]$$

$$\Leftrightarrow \mathcal{F}\{x(t+1)\} = X(\omega)e^{j\omega} = e^{j\omega} \text{rect}\left[\frac{\omega-1}{2}\right]$$

d-) $\mathcal{F}\{x(-2t+4)\} = \mathcal{F}\{x[-2(t-2)]\}$

$$= \frac{1}{2} e^{j2\omega} X\left(\frac{-\omega}{2}\right)$$

$$= \frac{1}{2} e^{-j2\omega} \text{rect}\left[\frac{\omega+2}{4}\right]$$

(f) $\mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega X(\omega) = j\omega \text{rect}\left(\frac{\omega-1}{2}\right)$

4.8. $x(t) = A \text{rect}\left(\frac{t+a/2}{a}\right) - A \text{rect}\left(\frac{t-a/2}{a}\right)$

$$\mathcal{F}\{x(t)\} \Rightarrow \text{rect}\left(\frac{t}{\tau}\right) = \tau \text{sinc}\left\{\frac{\omega\tau}{2}\right\}$$

(Shifting property $\Rightarrow$) $x(t-\alpha) = X(\omega) \cdot e^{-j\omega\alpha}$

$$X(\omega) = A \cdot a \text{sinc}\left(\frac{\omega a}{2}\right) e^{-j\frac{a}{2}\omega} - A \cdot a \text{sinc}\left(\frac{\omega a}{2}\right) e^{j\frac{a}{2}\omega}$$

$$= A \cdot a \text{sinc}\left(\frac{\omega a}{2}\right) \cdot \left(e^{-j\frac{a}{2}\omega} - e^{j\frac{a}{2}\omega}\right)$$

$$X(\omega) = A \cdot a \cdot \text{sinc}\left(\frac{\omega a}{2}\right) \left(2j \sin\left(\frac{\omega a}{2}\right) \right) \quad (9)$$

$$y(t) = A \Delta\left(\frac{t}{a}\right)$$

$$Y(\omega) = A \cdot a \cdot \text{sinc}^2 \frac{\omega a}{2\pi}$$

4.15. a) $x(-2t+1)$

$$x(-2t+1) = x(-2(t-\frac{1}{2}))$$

$$\mathcal{F}\{x(-2(t-\frac{1}{2}))\} = \frac{1}{|-2|} X\left(\frac{\omega}{-2}\right) e^{-j\omega(\frac{1}{2})}$$

$$= \frac{1}{2} e^{-j\omega/2} \cdot \frac{\frac{\omega^2}{4} - j2\omega + 2}{-\frac{\omega^2}{4} - j2\omega + 3}$$

b-1 $x(t) e^{-j\omega t}$

$$\mathcal{F}\{x(t) \cdot e^{-j\omega t}\} = X(\omega - (-1))$$

$$= X(\omega + 1)$$

$$= \frac{(w+1)^2 + j4(w+1) + 2}{-(w+1)^2 + j4(w+1) + 3} //$$