

EE3302-501-HW5 Solutions

1 Oppenheim Book

2.3. Let us define the signals

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_1[n] = u[n].$$

We note that

$$x[n] = x_1[n-2] \quad \text{and} \quad h[n] = h_1[n+2].$$

Now,

$$\begin{aligned} y[n] &= x[n] * h[n] = x_1[n-2] * h_1[n+2] \\ &= \sum_{k=-\infty}^{\infty} x_1[k-2] h_1[n-k+2]. \end{aligned}$$

By replacing k with $m+2$ in the above summation, we obtain

$$y[n] = \sum_{m=-\infty}^{\infty} x_1[m] h_1[n-m] = x_1[n] * h_1[n].$$

Using the results of Example 2.1 in the textbook, we may write:

$$y[n] = 2 \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] u[n]$$

2.5. The signal $y[n]$ is

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

In this case, this summation reduces to

$$y[n] = \sum_{k=0}^9 x[k] h[n-k] = \sum_{k=0}^9 h[n-k]$$

From this, it is clear that $y[n]$ is a summation of shifted replicas of $h[n]$. Since the last replica will begin at $n = 9$ and $h[n] = 0$ for $n > N$, it follows that

$$y[n] = 0 \quad \text{for } n > N + 9.$$

Using this and the fact that $y[14] = 0$, we conclude that

$$N \leq 4.$$

Furthermore, since $y[4] = 5$, we conclude that $h[n]$ has at least 5 non-zero points. Therefore, the only value of N that satisfies both conditions is

$$N = 4.$$

2.6. From the given information, we have:

$$\begin{aligned} y[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^{-k} u[-k-1]u[n-k-1] \\ &= \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k} u[n-k-1] \\ &= \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k u[n+k-1] \end{aligned}$$

Replacing k by $p-1$,

$$y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} u[n+p]$$

For $n \geq 0$, the above equation becomes:

$$y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \frac{1/3}{1-1/3} = \frac{1}{2}$$

For $n < 0$, the equation reduces to:

$$\begin{aligned} y[n] &= \sum_{p=-n}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \left(\frac{1}{3}\right)^{-n+1} \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^p \\ &= \left(\frac{1}{3}\right)^{-n+1} \frac{1}{1-1/3} = \left(\frac{1}{3}\right)^{-n} \cdot \frac{1}{2} = \frac{3^n}{2} \end{aligned}$$

Therefore,

$$y[n] = \begin{cases} \frac{3^n}{2}, & n < 0 \\ \frac{1}{2}, & n \geq 0 \end{cases}$$

2.21. (a) The convolution is:

$$y[n] = x[n] * h[n]$$

$$\begin{aligned}
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \beta^n \sum_{k=0}^n (\alpha/\beta)^k \text{ for } n \geq 0 \\
&= \left[\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right] u[n] \text{ for } \alpha \neq \beta
\end{aligned}$$

(b) Using (a):

$$y[n] = \alpha^n \left[\sum_{k=0}^n 1 \right] u[n] = (n+1)\alpha^n u[n]$$

2.28. (a) Causal because $h[n] = 0$ for $n < 0$. Stable because $\sum_{n=0}^{\infty} (\frac{1}{5})^n = 5/4 < \infty$.

(b) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n=-2}^{\infty} (0.8)^n = 5 < \infty$.

(d) Not causal because $h[n] \neq 0$ for $n < 0$. Stable because $\sum_{n=-\infty}^3 5^n = \frac{625}{4} < \infty$.

(e) Causal because $h[n] = 0$ for $n < 0$. Unstable because the second term becomes infinite as $n \rightarrow \infty$.

2 Soliman Book

6.3. (a)

$$\frac{\Omega_0}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8}; \quad N = 8$$

(d) Since it is a power of e , it is not periodic.

6.4.

$$x(n) = x_0(nT) = 5 \cos \left(120nT - \frac{\pi}{3} \right)$$

$$\Omega_0 = 120T, \quad N = \frac{2\pi m}{120T} = \frac{\pi m}{60T}$$

Periodic if T is multiples of π .

6.5.

$$x(n) = 3 \sin(100n\pi T) + 4 \cos(120n\pi T)$$

$$N_1 = \frac{2\pi m}{100\pi T} = \frac{m}{50T}, \quad N_2 = \frac{2\pi l}{120\pi T} = \frac{l}{60T} \quad \text{for any integer } T.$$

$$N = pN_1 = qN_2 \quad \text{so that} \quad p = \frac{5ql}{6m}$$

Periodic for any integer T .

6.8. (i)

$$y(n) = \sum_{k=-\infty}^n x(k) = \begin{cases} 2(-n-6) & -5 \leq n \leq -1 \\ 2(n-4) & 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(ii)

$$\begin{aligned} y(n) &= \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1) + \left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^n u(n) \\ &= \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1) + 6 \left[\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{3}\right)^{n+1} \right] u(n) \\ &= 4 \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1) - 2 \left(\frac{1}{3}\right)^n u(n) \end{aligned}$$

(iv)

$$\begin{aligned} y(n) &= \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{3}\right)^n * \left(\frac{1}{3}\right)^n u(n) \\ &= \left(\frac{1}{3}\right)^n u(n) + (n+1) \left(\frac{1}{3}\right)^n \end{aligned}$$

6.10. (a)

$$\begin{aligned} h(n) &= [h_1(n) * h_2(n) - h_4(n)] * h_3(n) \\ &= \left[(n+1) \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n) \right] * u(n) \\ &= \sum_{k=0}^n (k+1) \left(\frac{1}{3}\right)^k - \sum_{k=0}^n \left(\frac{1}{2}\right)^k \\ &= -\frac{1}{2}n \left(\frac{1}{3}\right)^n u(n) - \frac{5}{4} \left(\frac{1}{3}\right)^n u(n) + \frac{9}{4}u(n) - 2u(n) + \left(\frac{1}{2}\right)^n u(n) \\ &= -\frac{1}{2}n \left(\frac{1}{3}\right)^n u(n) - \frac{5}{4} \left(\frac{1}{3}\right)^n u(n) + \frac{1}{4}u(n) + \left(\frac{1}{2}\right)^n u(n) \end{aligned}$$

(b)

$$\begin{aligned} s(n) &= \sum_{k=0}^n h(k) = \sum_{k=0}^n \left[-\frac{1}{2}k \left(\frac{1}{3}\right)^k - \frac{5}{4} \left(\frac{1}{3}\right)^k + \frac{1}{4} + \left(\frac{1}{2}\right)^k \right] \\ &= \left[-\frac{1}{6} \left(\frac{1}{3}\right)^n + \frac{5}{24} \left(\frac{1}{3}\right)^n - \left(\frac{1}{2}\right)^n + \frac{1}{4}n + \frac{9}{8} \right] u(n) \end{aligned}$$