

Homework 7 Solutions

Oppenheim Book

3.16. (a) The given signal $x_1[n]$ is

$$x_1[n] = (-1)^n = e^{j\pi n} = e^{j(2\pi/2)n}$$

Therefore, $x_1[n]$ is periodic with period $N=2$ and its Fourier series coefficients in the range $0 \leq k \leq 1$ are

$$a_0 = 0, \text{ and } a_1 = 1$$

Using the results derived in Section 3.8 in the Oppenheim book, the output $y_1[n]$ is given by

$$\begin{aligned} y_1[n] &= \sum_{k=0}^1 a_k H(e^{j2\pi k/2}) e^{j(2\pi/2)kn} \\ &= 0 + a_1 H(e^{j\pi}) e^{j\pi n} = 0 \end{aligned}$$

(b) The signal $x_2[n]$ is periodic with period $N=16$. The signal $x_2[n]$ may be written as

$$\begin{aligned} x_2[n] &= e^{j(2\pi/16)(0)n} - (j/2) e^{j(\pi/4)} e^{j(2\pi/16)(3)n} + (j/2) e^{j\pi/4} e^{j(2\pi/16)(3)n} \\ &= e^{j(2\pi/16)(0)n} - (j/2) e^{j(\pi/4)} e^{j(2\pi/16)(3)n} + (j/2) e^{j\pi/4} e^{j(2\pi/16)(3)n} \end{aligned}$$

Therefore, the non-zero Fourier series coefficients of $x_2[n]$ in the range $0 \leq k \leq 15$ are

$$a_0 = 1, \quad a_3 = -(\sqrt{2}/2) e^{j(\pi/4)} \quad a_{13} = (\sqrt{2}/2) e^{-j(\pi/4)} \quad (2)$$

The output $y_2[n]$ is given by

$$\begin{aligned} y_2[n] &= \sum_{k=0}^{15} a_k H(e^{j2\pi k/16}) e^{j(2\pi k/16)n} \\ &= 0 - (\sqrt{2}/2) e^{j(\pi/4)} e^{j(2\pi/16)(3)n} + (\sqrt{2}/2) e^{-j(\pi/4)} e^{j(2\pi/16)(13)n} \\ &= \sin\left(\frac{3\pi}{8}n + \frac{\pi}{4}\right) \end{aligned}$$

3.28. (b) $x[n] = \sin\left(\frac{2\pi n}{3}\right) \cdot \cos\left(\frac{\pi n}{2}\right)$

$$x[n] = \frac{1}{2} \left(\sin\left(\frac{2\pi n}{3} + \frac{\pi n}{2}\right) + \sin\left(\frac{2\pi n}{3} - \frac{\pi n}{2}\right) \right)$$

$$x[n] = \frac{1}{2} \left(\sin\left(\frac{7\pi}{6}n\right) + \sin\left(\frac{\pi}{3}n\right) \right)$$

$$N = 12$$

$$x[n] = \frac{1}{2} \left[\frac{1}{2j} \left(e^{j(2\pi/12) \cdot 7n} - e^{-j(2\pi/12) \cdot 7n} \right) + \frac{1}{2j} \left(e^{j(2\pi/12)n} - e^{-j(2\pi/12)n} \right) \right]$$

Therefore:

$$a_1 = \frac{1}{4j} = a_{11}^*$$

$$a_5 = -\frac{1}{4j} = a_7^*, \quad a_k = 0 \text{ otherwise}$$

The plot of magnitude and phase in page 5.

3.28. (c) $x[n] = 1 - \sin \frac{\pi n}{4} \quad 0 \leq n \leq 3$

(3)

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j k (2\pi/N)n}$$

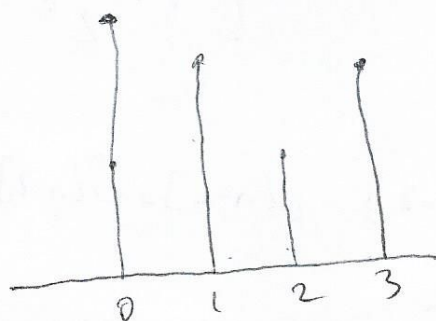
$$= \frac{1}{4} \sum_{n=0}^3 \left(1 - \sin \frac{\pi n}{4} \right) e^{-j \frac{2\pi}{4} k n}$$

$$= \frac{1}{4} \left[(1-0) e^{-j0} + \left(1 - \sin \frac{\pi}{4} \right) e^{-j \frac{2\pi}{4} k} + \left(1 - \sin \frac{\pi}{2} \right) e^{-j \frac{2\pi}{4} 2k} \right. \\ \left. + \left(1 - \sin \frac{3\pi}{4} \right) e^{-j \frac{2\pi}{4} 3k} + \left(1 - \sin \pi \right) e^{-j \frac{2\pi}{4} 4k} \right]$$

$$= \frac{1}{4} \left[1 + \left(1 - \frac{1}{\sqrt{2}} \right) e^{-j(\pi - \frac{\pi}{2})k} + \left(1 - \frac{1}{\sqrt{2}} \right) e^{-j(\pi + \frac{\pi}{2})k} \right]$$

Therefore ;

$$= \frac{1}{4} \left[1 + 2(-1)^k \left(1 - \frac{1}{\sqrt{2}} \right) \cos \left(\frac{\pi}{2} k \right) \right]$$



$|a_k|$

$\angle a_k$



3.37. (b) The Fourier series coefficients of $x[n]$ are: (4)

$$a_k = \frac{1}{6} [1 + 2\cos(k\pi/3)], \text{ for all } k.$$

Also $N=6$. Therefore, the Fourier series coefficients of $y[n]$ are:

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} - \frac{1}{1 - 2e^{-j\omega}}$$

$$b_k = a_k H(e^{j2k\pi/N}) = \frac{1}{6} [1 + 2\cos(k\pi/3)] \cdot H(e^{j2k\pi/N})$$

$$b_k = \frac{1}{6} [1 + 2\cos(k\pi/3)] \left[\frac{1}{1 - \frac{1}{2}e^{-j\pi k/3}} - \frac{1}{1 - 2e^{-j\pi k/3}} \right]$$

5.21. (a) $x[n] = u[n-2] - u[n-6] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$

Using the Fourier transform analysis eq. (5.9) in Oppen-
heim book, we obtain

$$X(e^{j\omega}) = e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega} //$$

$$5.21. (d) X(e^{j\omega}) = \sum_{n=-\infty}^0 2^n \sin\left(\frac{\pi n}{4}\right) e^{-j\omega n}$$

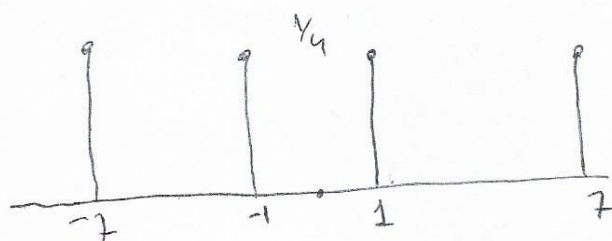
(5)

$$= - \sum_{n=0}^{\infty} 2^{-n} \sin\left(\frac{\pi n}{4}\right) e^{j\omega n}$$

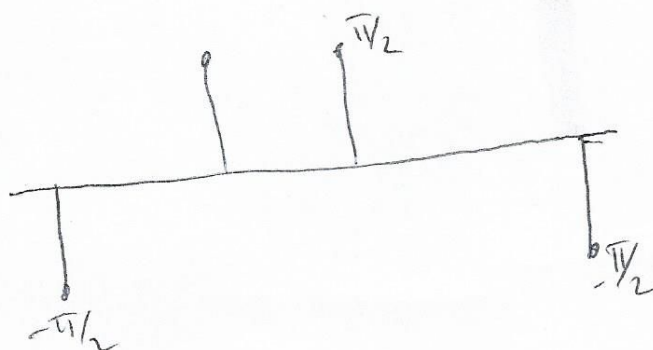
$$= - \frac{1}{2j} \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n e^{j\pi n/4} e^{j\omega n} - \left(\frac{1}{2}\right)^n e^{-j\pi n/4} e^{j\omega n} \right]$$

$$= - \frac{1}{2j} \left[\frac{1}{1 - (\frac{1}{2}) e^{j\pi/4} e^{j\omega}} - \frac{1}{1 - (\frac{1}{2}) e^{-j\pi/4} e^{j\omega}} \right]$$

3.28. (b)



$|a_k|$



$\angle a_k$

(6)

Solimen book

$$7.1. (b) \quad x(n) = 4 \cos \frac{2\pi n}{3} \sin \frac{\pi n}{3}$$

$$x(n) = 2 \left[\sin \frac{7\pi n}{6} - \sin \frac{\pi n}{6} \right]$$

$$N=12, \quad \Omega_0 = \frac{\pi}{6}$$

$$x(n) = \frac{1}{j} \left[e^{j7\Omega_0 n} - e^{-j7\Omega_0 n} - e^{j\Omega_0 n} + e^{-j\Omega_0 n} \right]$$

$$a_1 = -\frac{1}{j} = j = a_{-1}^* = a_{11}^*$$

$$a_7 = -j = a_{-7}^* = a_5^*$$

All others zero

$$(c) \quad x(n) = \{1, -1, 0, 1, -1\}, \quad N=5$$

$$e^{j\frac{2\pi}{5}} = 0.3090 + j 0.9511$$

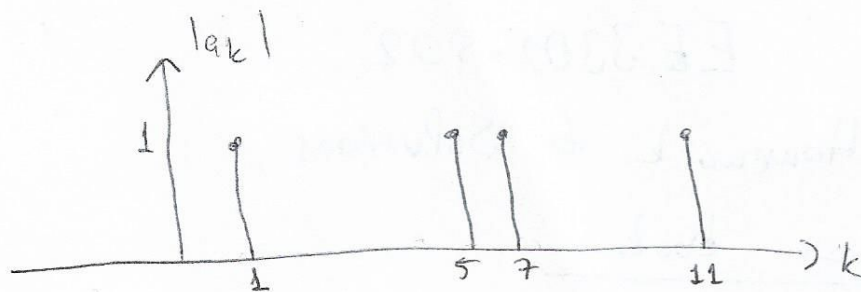
$$a_k = \frac{1}{5} \left[1 - e^{j\frac{2\pi}{5}k} + e^{j\frac{6\pi}{5}k} - e^{j\frac{8\pi}{5}k} \right]$$

$$a_0 = 0 \quad a_1 = -0.4271 - j 0.5878 = a_4^*$$

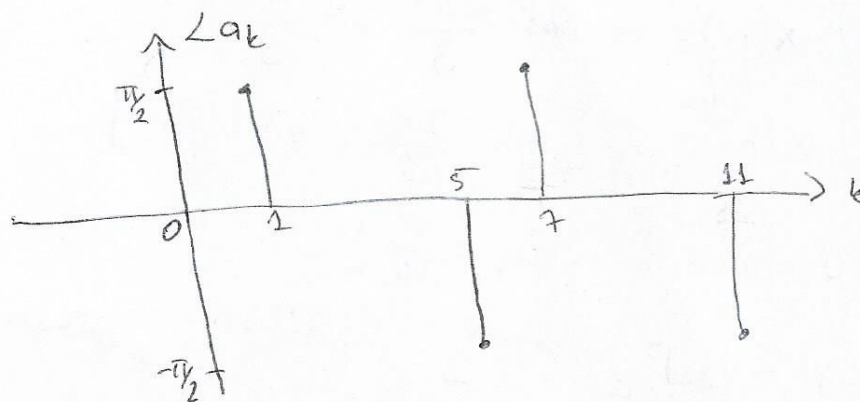
$$a_2 = 2.9271 + j 0.9511 = a_3^*$$

7.1.(b)

Plot

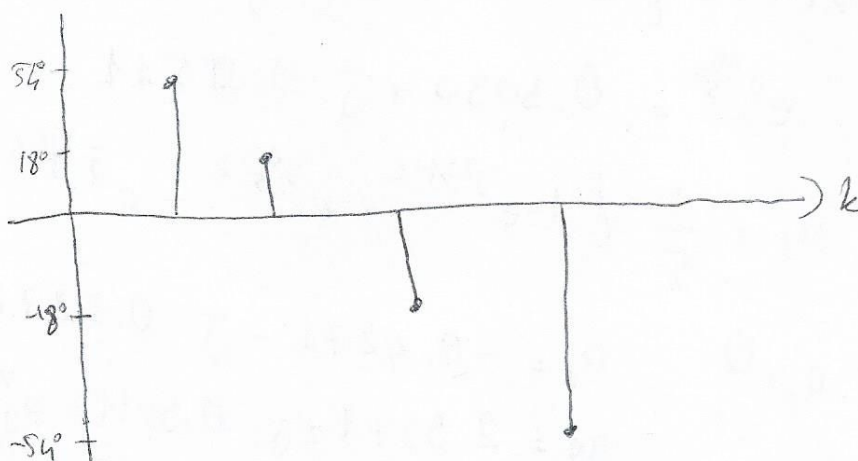
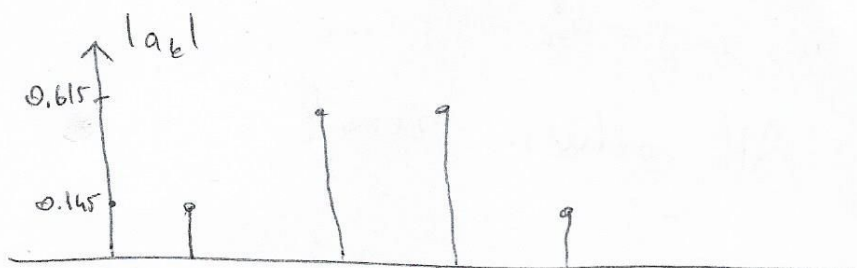


(7)



7.1.(c)

Plot



$$7.2. (a) \quad \Omega_0 = \frac{\pi}{4}$$

(8)

$$a_k = 1 + \frac{1}{2} \cos 2\Omega_0 k + 2 \cos \Omega_0 k$$

$$= 1 + \frac{1}{4} e^{-j2\Omega_0 k} + \frac{1}{4} e^{-j6\Omega_0 k} + e^{-j\Omega_0 k} + e^{-j7\Omega_0 k}$$

$$x(0) = 8, \quad x(1) = x(7) = 8, \quad x(2) = x(6) = 2 \\ x(3) = x(4) = x(5) = 0$$

$$(b) \quad a_k = \{1, 1, 1, 1, 0, 0, 0, 0\}$$

$$x(n) = 1 + e^{j\frac{2\pi}{8}n} + e^{j\frac{2\pi}{8}2n} + e^{j\frac{2\pi}{8}3n}$$

$$x(0) = 4 \quad x(1) = 1 + j \cdot 2.4142 \quad x(2) = 0$$

$$x(3) = 1 + j \cdot 2.4142 \quad x(4) = 0 \quad x(5) = 1 - j \cdot 0.4142$$

$$x(6) = 0 \quad x(7) = 1 - j \cdot 2.4142$$

$$7.22. (a) \quad X(\Omega) = \sum_{n=0}^{\infty} e^{-j\Omega n} = \frac{1 - e^{-j\Omega(n_0+1)}}{1 - e^{-j\Omega}} \\ = \frac{\sin \frac{\Omega(n_0+1)}{2}}{\sin \frac{\Omega}{2}} e^{j\Omega \frac{n_0}{2}}$$

$$7.12. (b) \mathcal{F} \left\{ \left(\frac{1}{3} \right)^{|n|} \right\} = \sum_{n=-\infty}^{-1} \left(\frac{1}{3} \right)^{-n} e^{-j\Omega n} + \sum_{n=0}^{\infty} \left(\frac{1}{3} \right)^n e^{-j\Omega n}$$

$$= \frac{\frac{1}{3} e^{j\Omega}}{1 - \frac{1}{3} e^{j\Omega}} + \frac{1}{1 - \frac{1}{3} e^{-j\Omega}} = \frac{4}{5 - 3 \cos \Omega}$$

(9)

$$\mathcal{F} \left\{ n \left(\frac{1}{3} \right)^{|n|} \right\} = j \frac{d}{d\Omega} \frac{4}{5 - 3 \cos \Omega}$$

$$= j \frac{4 \sin \Omega}{(5 - 3 \cos \Omega)^2}$$

$$(9) X(\Omega) = \sum_{n=0}^{N_0-1} \alpha^n e^{-j\Omega n} = \frac{1 - (\alpha e^{-j\Omega})^{N_0}}{1 - \alpha e^{-j\Omega}} //$$

$$7.13. (a) \sum_{k=0}^{N-1} a_k e^{j k \Omega_0 n} \longleftrightarrow \sum_{k=0}^{N-1} 2\pi a_k \delta(\Omega - k \Omega_0)$$

$$x(n) = \frac{j}{2} e^{j \frac{\pi}{3} n} + \frac{1}{2} e^{j \frac{2\pi}{3} n} + \frac{1}{2} e^{j \frac{4\pi}{3} n} + \frac{j}{2} e^{j \frac{5\pi}{3} n}$$

$$= \cos \frac{2\pi n}{3} - \frac{1}{2} \sin \frac{\pi n}{3}$$

$$7.13(b) \quad X(\Omega) = 4 \sin 5\Omega + 2 \cos \Omega \quad (10)$$

$$= 4 \frac{(e^{j5\Omega} - e^{-j5\Omega})}{2j} + \frac{2(e^{j\Omega} + e^{-j\Omega})}{2}$$

$$= -j2\delta(n+5) + j\delta(n-5) + \delta(n-1) + \delta(n+1) //$$

A-1- $y(n) - \frac{1}{4}y(n-2) = x(n)$ and

(11)

$$X(n) = 3 + 5 \cos[\pi n/4] - \sin[\pi(n-3)/2]$$

i) $h(n) = ?$

$$Y(\Omega) = \frac{1}{4} Y(\Omega) e^{-j2\Omega} = X(\Omega)$$

$$Y(\Omega) \left(1 - \frac{1}{4} e^{-j2\Omega}\right) = X(\Omega)$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1 - \frac{1}{4} e^{-j2\Omega}}$$

$$H(\Omega) = \frac{1}{2} \left[\frac{1}{1 - \frac{1}{2} e^{-j\Omega}} + \frac{1}{1 + \frac{1}{2} e^{-j\Omega}} \right]$$

$$h(n) = \frac{1}{2} \left[\left(-\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n \right] u(n) //$$

ii) From lecture notes chapter 7 page 22

$$y(n) = 3 |H(\Omega=0)| + 5 |H(\Omega=\frac{\pi}{4})| \cos(\pi n/4) - |H(\Omega=\frac{\pi}{2})| \sin\left[\frac{\pi(n-3)}{2}\right]$$

$$H(\Omega=0) = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

(12)

$$H(\Omega=\frac{\pi}{4}) = \frac{1}{1-\frac{1}{4}e^{-j2\pi\frac{1}{4}}} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$H(\Omega=\frac{\pi}{3}) = \frac{1}{1-\frac{1}{4}e^{-j2\pi\frac{1}{3}}}$$

Lecture Notes
Chapter 7
page 23 //

$$|H(\Omega=\frac{\pi}{3})| = \frac{1}{\sqrt{1-2\cdot\frac{1}{4}\cdot\cos\frac{2\pi}{3}+\frac{1}{16}}} = \frac{4\sqrt{21}}{21}$$

$$y(n) = 3 \cdot \frac{4}{3} + 5 \cdot \frac{4}{3} \cdot \cos\left(\frac{\pi n}{4}\right) - \frac{4}{21} \sqrt{21} \sin\left[\frac{\pi(n-3)}{2}\right]$$

$$y(n) = 4 + \frac{20}{3} \cos\left(\frac{\pi n}{4}\right) - \frac{4}{21} \sqrt{21} \sin\left[\frac{\pi(n-3)}{2}\right] //$$

A-2- $y(n) + \frac{1}{2} y(n-1) = x(n)$

(13)

$$Y(\Omega) + \frac{1}{2} Y(\Omega) \cdot e^{-j\Omega} = X(\Omega)$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1 + \frac{1}{2} e^{-j\Omega}}$$

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$x(n) = \left(-\frac{1}{2}\right)^n u(n)$$

$$\hookrightarrow X(\Omega) = \frac{1}{1 - \frac{1}{2} e^{-j\Omega}}$$

$$Y(\Omega) = H(\Omega) \cdot X(\Omega)$$

$$= \frac{1}{1 + \frac{1}{2} e^{-j\Omega}} \cdot \frac{1}{1 - \frac{1}{2} e^{-j\Omega}} = \frac{1}{1 - \frac{1}{4} e^{-j2\Omega}}$$

$Y(\Omega)$ can be written as following:

$$Y(\Omega) = \frac{1}{2} \left(\frac{1}{1 + \frac{1}{2} e^{-j\Omega}} + \frac{1}{1 - \frac{1}{2} e^{-j\Omega}} \right)$$

$$\hookrightarrow y(n) = \frac{1}{2} \left\{ \left(\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^n \right\} u(n) //$$