

Homework - 1 Solutions

Soliman Book:

1.5. $\exp[j\omega t] = \cos \omega t + j \sin \omega t$ (Euler's formula)
 Since $\cos \omega t$ and $\sin \omega t$ are periodic with period $\frac{2\pi}{\omega}$, then their linear combination is also periodic with period $\frac{2\pi}{\omega}$.

1.6. a-) $x(t) = \underbrace{\sin\left(\frac{\pi}{3}t\right)}_{T_1} + 2 \underbrace{\cos\left(\frac{8\pi}{3}t\right)}_{T_2}$

$$T_1 = 6, \quad T_2 = \frac{3}{4} \Rightarrow \frac{T_1}{T_2} = 8.$$

Periodic, with period $T=6$.

b-) $x(t) = \underbrace{\exp\left[j\frac{7\pi}{6}t\right]}_{T_1} + \underbrace{\exp\left[j\frac{5\pi}{6}t\right]}_{T_2}$

$$T_1 = \frac{2\pi}{\frac{7\pi}{6}} = \frac{12}{7}$$

$$T_2 = \frac{2\pi}{\frac{5\pi}{6}} = \frac{12}{5}$$

$$\frac{T_1}{T_2} = \frac{\frac{12}{7}}{\frac{12}{5}} = \frac{5}{7} \Rightarrow \text{Periodic, with period } T=12$$

1.8. b.) $x(t) = A[u(t-a) - u(t+a)]$

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$$\text{Energy} = A^2 \int_{-\infty}^{\infty} [u(t-a) - u(t+a)]^2 dt$$

$$= A^2 \int_{-a}^a dt = A^2 \cdot 2a < \infty$$

$$\text{Power} = \lim_{T \rightarrow \infty} \frac{1}{2T} A^2 \int_{-T}^T [u(t-a) - u(t+a)]^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-a}^a dt = 0 //$$

So, it is an energy signal.

g-) $x(t) = A \cdot \exp[bt]$, $b > 0$

$$\text{Energy} = A^2 \int_{-\infty}^{\infty} \exp[2bt] dt$$

$$= A^2 \left[\frac{\exp[2bt]}{2b} \right]_{-\infty}^{\infty} = \infty$$

$$\text{Power} = \lim_{T \rightarrow \infty} \frac{1}{2T} A^2 \int_{-T}^T \exp[2bt] dt$$

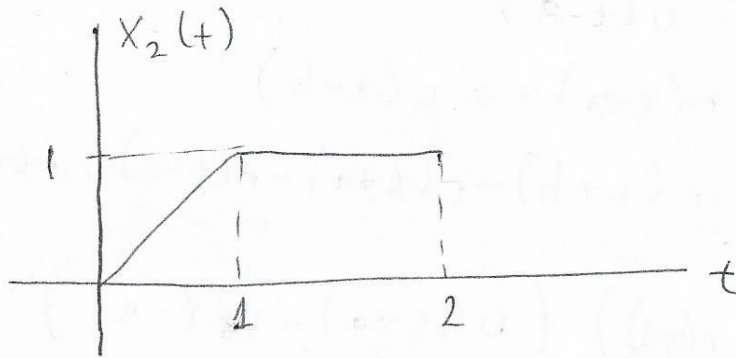
$$= \lim_{T \rightarrow \infty} \frac{1}{2T} A^2 \cdot \left[\frac{\exp[2bt]}{2b} \right]_{-T}^T$$

= Undefined

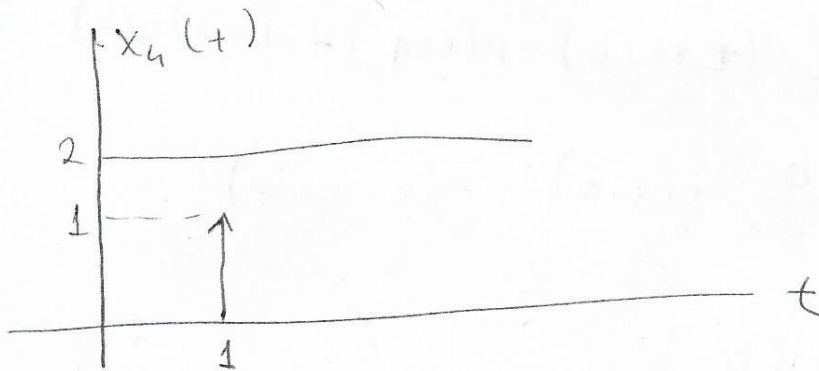
So, $x(t)$ is neither energy nor power signal.

(3)

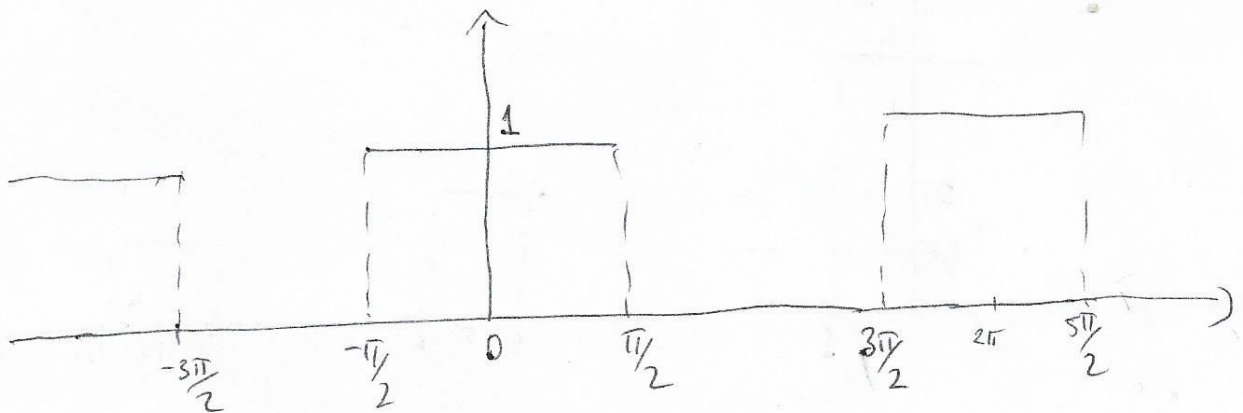
$$1-13-b) \quad x_2(t) = r(t) - r(t-1) - u(t-2)$$



$$d-) \quad x_4(t) = 2u(t) + \delta(t-1)$$



$$g-) \quad x_7(t) = u \cos(t) = \begin{cases} 1, & \cos t > 0 \\ 0, & \text{otherwise} \end{cases}$$



(4)

1.16. $x_1(t) = v(t) - v(t-a)$

$$x_2(t) = r(t) - r(t-a) - a v(t-b)$$

$$x_3(t) = \frac{a}{b-a} [r(t+b) - r(t+a) - r(t-a) + r(t-b)]$$

$$x_4(t) = (r(t) - r(t-a)) (v(t+a) - v(t-a))$$

$$x_5(t) = \frac{1}{c} r(t) - \frac{a}{c(a-c)} r(t-c) + \frac{1}{c} r(t-2a+c)$$

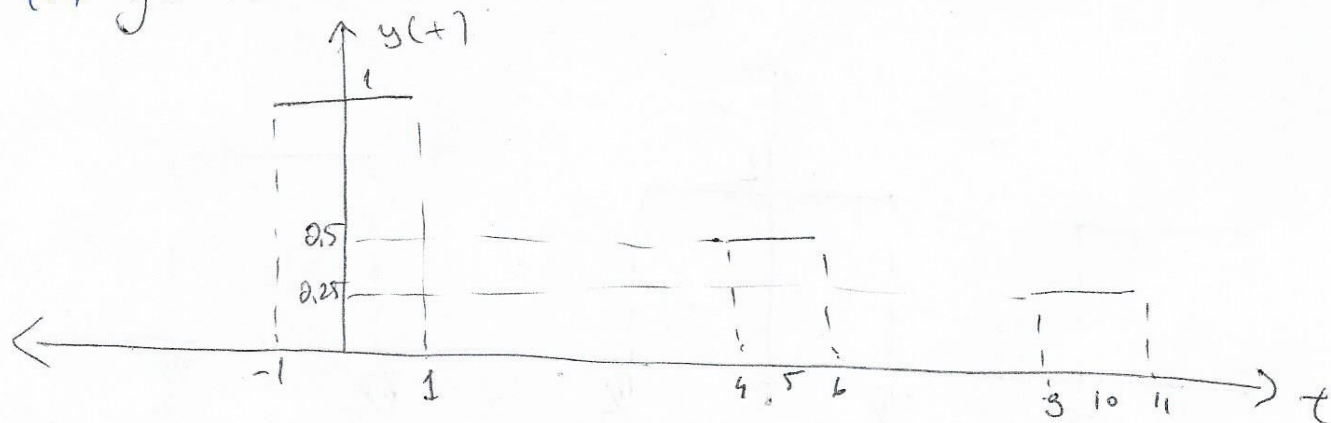
$$x_6(t) = \frac{a}{b-a} [r(t+a+b) - r(t+a)] + (b-a)v(t) \dots$$

$$- \frac{b}{b-a} r(t-a) + r(t-a-b)$$

1.18. $x(t) = \text{rect}(t/2)$

$$y(t) = x(t) + 0.5 x(t-5) + 0.25 x(t-10) \quad T \gg 2$$

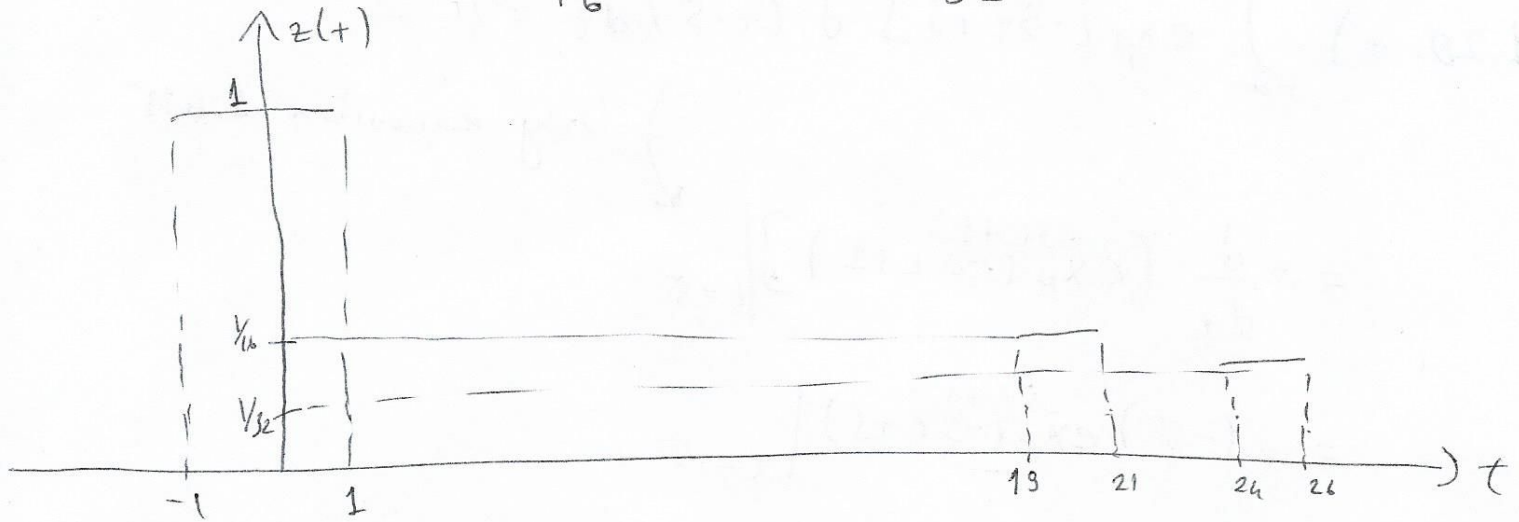
(a) $y(t) = x(t) + 0.5 x(t-5) + 0.25 x(t-10)$



(b) $z(t) = y(t) + 0.5 y(t-5) + 0.25 y(t-10)$

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$$z(t) = x(t) + \frac{1}{16} x(t-20) + \frac{1}{32} x(t-25)$$



1.20.1 b-) $\int_{-\infty}^{\infty} (t-1) \delta\left(\frac{2}{3}t - \frac{3}{2}\right) dt = ?$

Scaling property

$$= \int_{-\infty}^{\infty} (t-1) \frac{3}{2} \delta\left(t - \frac{3/2}{3/2}\right) dt$$

$$t = \frac{9}{4}$$

$$= \int_{-\infty}^{\infty} \left(\frac{9}{4} - 1\right) \frac{3}{2} \delta\left(t - \frac{9}{4}\right) dt$$

$$= \frac{15}{8} //$$

a. $\int_{-\infty}^{\infty} \left(\frac{2}{3}t - \frac{3}{2}\right) \delta\left(t - 1\right) dt = \frac{2}{3} - \frac{3}{2} = -\frac{5}{6}$

d. $\int_{-\infty}^{\infty} \left(e^{-t} + 1\right) + \sin\left(\frac{2\pi}{3}t\right) \delta\left(t - \frac{3}{2}\right) dt = e^{-0.5} + \sin(\pi)$
 $= e^{-0.5}$

(6)

$$1.20. e) \int_{-\infty}^{\infty} \exp[-5t+1] \delta'(t-5) dt = ?$$

Using equation 1.6.15.

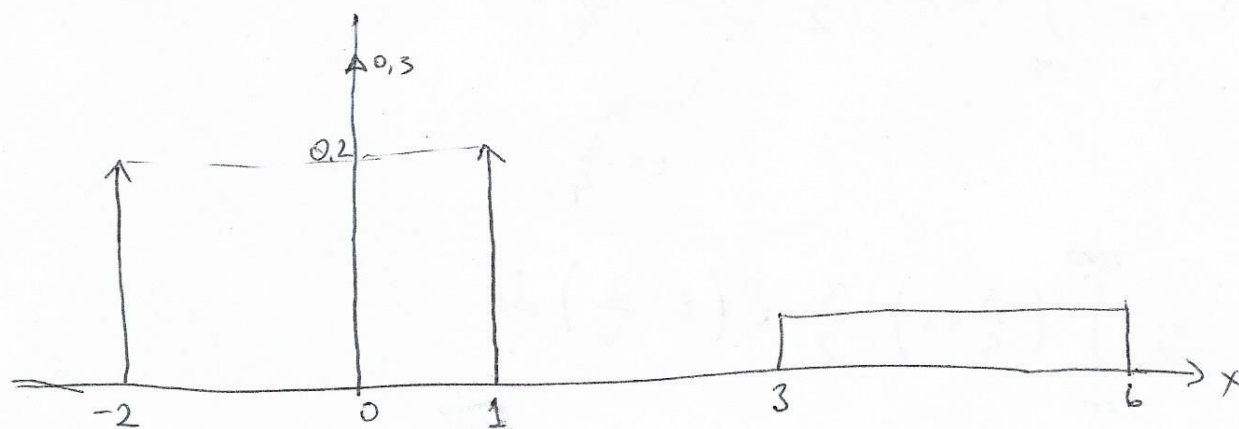
$$= - \frac{d}{dt} [\exp(-5t+1)] \Big|_{t=5}$$

$$= -(-5) \exp(-5t+1) \Big|_{t=5}$$

$$= 5 \exp[-24] //$$

$$1.21. a) P[X \leq \infty] = \int_{-\infty}^{\infty} f(x) dx$$

$$f(x) = 0.2 \delta(x+2) + 0.3 \delta(x) + 0.2 \delta(x-1) + 0.1 [u(x-3) - u(x-6)] dx$$



$$P[X \leq -3] = \int_{-\infty}^{-3} f(x) dx = 0 //$$

Oppenheim book:

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1.3. a-) $x_1(t) = e^{-2t} u(t)$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x_1(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T |e^{-4t}| dt = 0$$

$$E = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_0^{\infty} |e^{-4t}| dt = \frac{1}{4}$$

b-) $x_2(t) = e^{j(2t + \frac{\pi}{4})}$

$$|x_2(t)| = |e^{j(2t + \frac{\pi}{4})}| = |\cos(2t + \frac{\pi}{4}) + j \sin(2t + \frac{\pi}{4})| = 1$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x_2(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T 1 dt = \frac{1}{2}$$

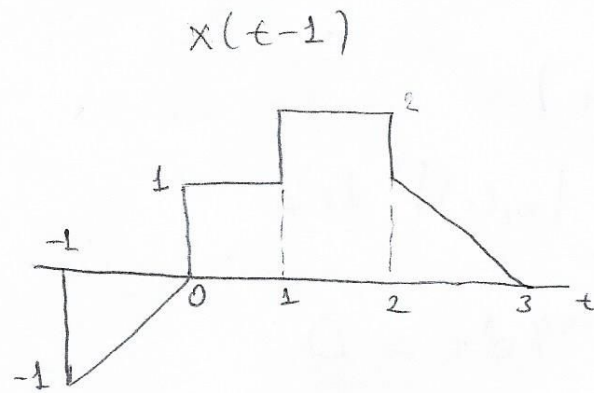
$$E = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} 1 dt = \infty$$

c-) $x_3(t) = \cos(t)$

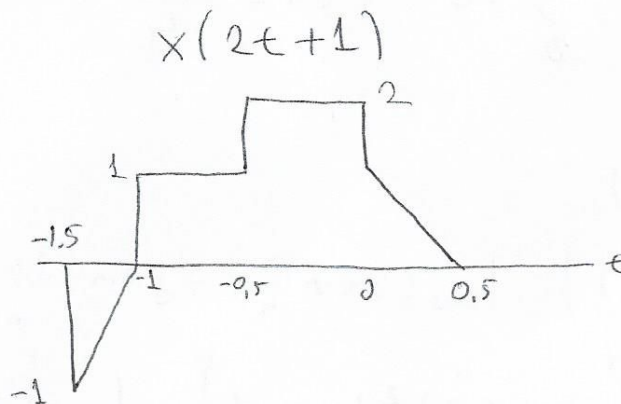
$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \cos^2(t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \left(\frac{1 + \cos(2t)}{2} \right) dt = \frac{1}{2}$$

$$E = \int_{-\infty}^{\infty} \cos^2(t) dt = \infty$$

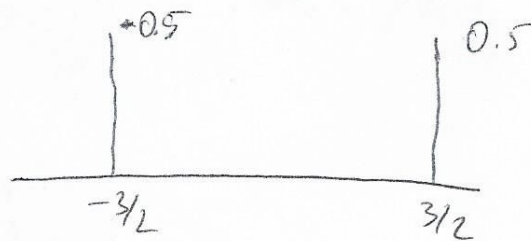
1.21. a-)



c-)



f-)

1.25. a-) $x(t) = 3 \cos(4t + \frac{\pi}{3})$

$$\frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2} \Rightarrow \text{Periodic}$$

(9)

1.25. b-) $x(t) = e^{j(\pi t - 1)}$

$$\frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2 \Rightarrow \text{Periodic} //$$

Fundamental period.

1.27. a-) $y(t) = x(t-2) + x(2-t)$

* $y_1(t) = x_1(t-2) + x_1(2-t)$

$y_2(t) = x_2(t-2) + x_2(2-t)$

$$y_3(t) = x_3(t-2) + x_3(2-t) = a x_1(t-2) + a x_1(2-t) + b x_2(t-2) + b x_2(2-t)$$

$$y_3(t) = a y_1(t) + b y_2(t) //$$

So, $y(t)$ is linear.

* If $x(t) < B$, $x(t-2)$ and $x(2-t) < B$.

So, $y(t) < 2B$ and $y(t)$ is stable.

b) $y(t) = \cos(3t) \cdot x(t)$

* $y_1(t) = \cos(3t) \cdot x_1(t)$

$y_2(t) = \cos(3t) \cdot x_2(t)$

$$y_3(t) = \cos(3t) \cdot x_3(t) = a \cos(3t) x_1(t) + b \cos(3t) x_2(t)$$

$$y_3(t) = a y_1(t) + b y_2(t)$$

Hence $y(t)$ is linear.

* If $x(t) < B$, $y(t) = \cos(3t) \cdot x(t) < B$.

Therefore, $y(t)$ is stable.

* $y(t)$ depends on only current values of $x(t)$.

So, $y(t)$ is memoryless and causal.

f-) $y(t) = x(t/3)$

* $y_1(t) = x_1(t/3)$

$y_2(t) = x_2(t/3)$

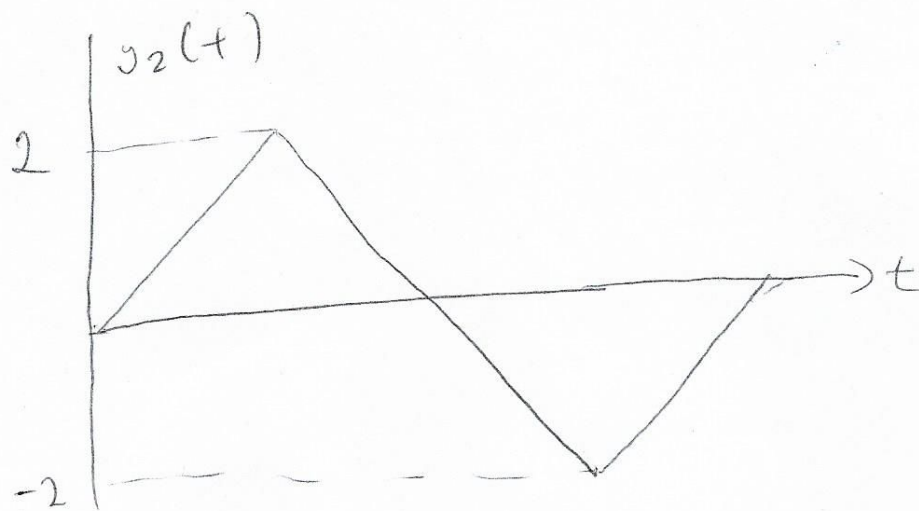
$y_3(t) = x_3(t/3) = a x_1(t/3) + b x_2(t/3)$

$y_3(t) = a y_1(t) + b y_2(t)$

Hence, $y(t)$ is linear.

* If $x(t) < B$, $y(t) < B$. Therefore
it is stable.

1.31. a-) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Hence, using
linearity we get $y_2(t) = y_1(t) - y_1(t-2)$.



b-) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$.

