

Homework-2 Solutions

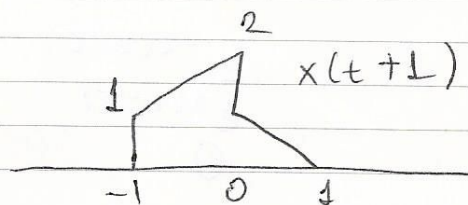
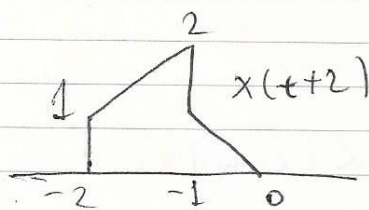
Oppenheim Book

2.8- Using the convolution integral,

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral reduces to

$$x(t) * h(t) = x(t+2) + 2x(t+1)$$



$$y(t) = \begin{cases} t+3, & -2 < t \leq -1 \\ t+4, & -1 < t \leq 0 \\ 2-2t, & 0 < t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(2)

2.9. Using the given definition for the signal $h(t)$, we may write,

$$h(\tau) = e^{2\tau} u(-\tau+4) + e^{-2\tau} u(\tau-5) = \begin{cases} e^{-2\tau}, & \tau > 5 \\ e^{2\tau}, & \tau < 4 \\ 0, & 4 < \tau < 5 \end{cases}$$

Therefore,

$$h(-\tau) = \begin{cases} e^{2\tau}, & \tau < -5 \\ e^{-2\tau}, & \tau > -4 \\ 0, & -5 < \tau < -4 \end{cases}$$

If we now shift the signal $h(-\tau)$ by t to the right, then the resultant signal $h(t-\tau)$ will be

$$\begin{cases} e^{-2(t-\tau)}, & \tau < t-5 \\ e^{2(t-\tau)}, & \tau > t-4 \\ 0, & (t-5) < \tau < (t-4) \end{cases}$$

Therefore

$$A = t-5 \quad B = t-4$$

2.11.(a) From the given information, we see that $h(t)$ is non zero only for $0 \leq t \leq \infty$. Therefore,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$= \int_0^{\infty} e^{-3\tau} (u(t-\tau-3) - u(t-\tau-5)) d\tau$$

(3)

We can easily show that $(u(t-\tau-3) - u(t-\tau-5))$ is non zero only in the range $(t-5) < \tau < (t-3)$.

Therefore, for $t \leq 3$, the above integral evaluates to zero. For $3 < t \leq 5$, the above integral is

$$y(t) = \int_0^{t-3} e^{-3\tau} d\tau = \frac{1 - e^{-3(t-3)}}{3}$$

For $t > 5$, the integral is

$$y(t) = \int_{t-5}^{t-3} e^{-3\tau} d\tau = \frac{(1 - e^{-6})e^{-3(t-5)}}{3}$$

Therefore, the result of this convolution may be expressed as :

$$y(t) = \begin{cases} 0, & -\infty < t \leq 3 \\ \frac{1 - e^{-3(t-3)}}{3}, & 3 < t \leq 5 \\ \frac{(1 - e^{-6})e^{-3(t-5)}}{3}, & 5 < t \leq \infty \end{cases}$$

(b) By differentiating $x(t)$ with respect to time we get

$$\frac{d x(t)}{d t} = \delta(t-3) - \delta(t-5)$$

Therefore,

$$g(t) = \frac{dx(t)}{dt} * h(t) = e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)$$

(c) From the result of part (a), we may compute the derivative of $y(t)$ to be

$$\frac{dy(t)}{dt} = \begin{cases} 0, & -\infty < t \leq 3 \\ e^{-3(t-3)}, & 3 < t \leq 5 \\ (e^{-6} - 1)e^{-3(t-5)}, & 5 < t < \infty \end{cases}$$

This is exactly equal to $g(t)$. Therefore, $g(t) = \frac{dy(t)}{dt}$.

2.14. (b) We determine if $h_2(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_2(\tau)| d\tau = \int_0^{\infty} e^{-\tau} |\cos(2\tau)| d\tau$$

This integral is clearly finite-valued because $e^{-\tau} |\cos(2\tau)|$ is an exponentially decaying function in the range $0 \leq \tau < \infty$

(5)

Therefore, $h_2(t)$ is the impulse response of a stable LTI system.

2.15. (b) We determine if $h_2[n]$ is absolutely summable as follows:

$$\sum_{k=-\infty}^{\infty} |h_2[k]| = \sum_{k=-\infty}^{\infty} 3^k \approx 3^{11/2}$$

Therefore, $h_2[n]$ is the impulse response of a stable LTI system.

2.22. (b). The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \\ &= \int_0^2 h(t-\tau) d\tau - \int_2^5 h(t-\tau) d\tau \end{aligned}$$

Therefore,

$$y(t) = \begin{cases} (1/2)[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t \leq 1 \\ (1/2)[e^2 + e^{2(t-5)} - 2e^{2(t-2)}], & 1 \leq t \leq 3 \\ (1/2)[e^{2(t-5)} - e^2], & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

$$\begin{aligned} 2.22 \text{ a)} \quad y(t) &= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \\ &= \int_0^t e^{-\alpha\tau} e^{-\beta(t-\tau)} d\tau, \quad t \geq 0 \\ &\rightarrow y(t) = \begin{cases} \frac{1}{\beta-\alpha} e^{-\beta t} (e^{-(\alpha-\beta)t} - 1) \cdot u(t) & \alpha \neq \beta \\ t e^{-\beta t} u(t) & \alpha = \beta \end{cases} \end{aligned}$$

(6)

2.29. (a) Causal because $h(t) = 0$ for $t < 0$.

Stable because $\sum_{n=-\infty}^{\infty} |h[n]| = 1 < \infty$.

(b) Not causal because $h(t) \neq 0$ for $t < 0$.

Unstable because $\int_{-\infty}^{\infty} |h(t)| dt = \infty$.

(d) Not causal because $h(t) \neq 0$ for $t < 0$.

Stable because $\int_{-\infty}^{\infty} |h(t)| dt = e^{-2}/2 < \infty$.

(e) Not causal because $h(t) \neq 0$ for $t < 0$.

Stable because $\int_{-\infty}^{\infty} |h(t)| dt = 1/3 < \infty$.

Soliman book:

2.3. (a) Note that $u(t+a) * u(t+b) = (t+b+a) u(t+b+a)$

$$y(t) = \text{rect}\left(\frac{t-a}{a}\right) * \delta(t-b)$$

$$= \text{rect}\left(\frac{t-a-b}{a}\right) //$$

$$(f) y(t) = t[u(t) - u(t-1)] * u(t)$$

$$= t u(t) * u(t) - t \cdot 1 u(t-1) * u(t) - u(t-1) * u(t)$$

$$= \frac{t^2}{2} u(t) - \frac{(t-1)^2}{2} u(t-1) - (t-1) u(t-1)$$

$$e) y(t) = u(t) * u(t) = t u(t) = \gamma(t)$$

(7)

$$(h) \quad y(t) = r(t) * [\operatorname{sgn}(t) + u(-t-1)]$$

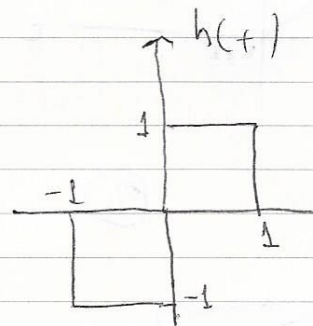
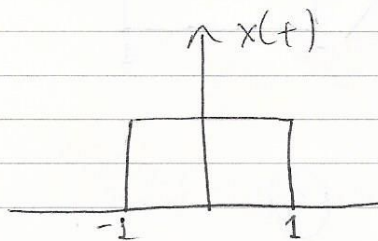
Note that;

$$\operatorname{sgn}(t) = 2u(t) - 1$$

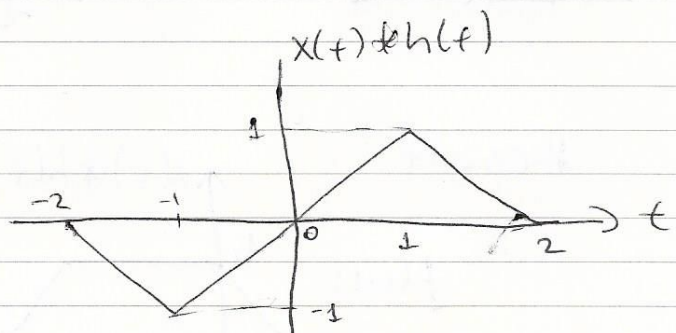
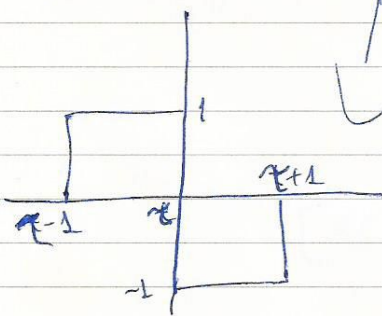
$$u(-t-1) = -u(t+1)$$

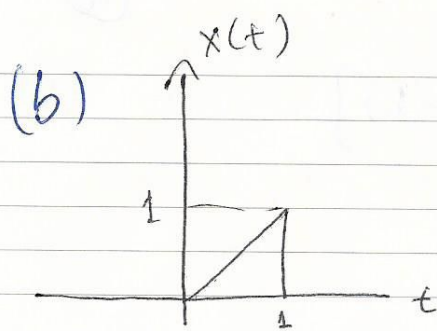
$$y(t) = -\frac{(t+1)^2}{2} u(t+1) + t^2 u(t)$$

2.4. (a)

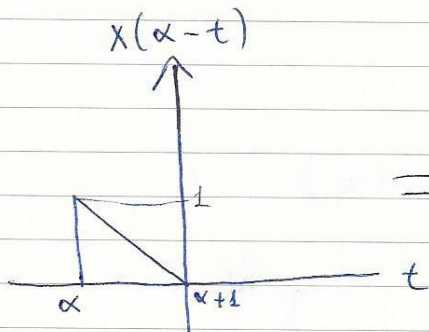


$h(-t+1)$

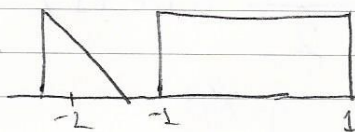




(8)



$\Rightarrow$



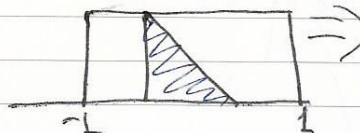
(2)

(3)

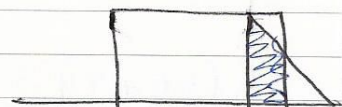
(4)



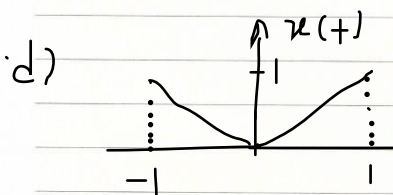
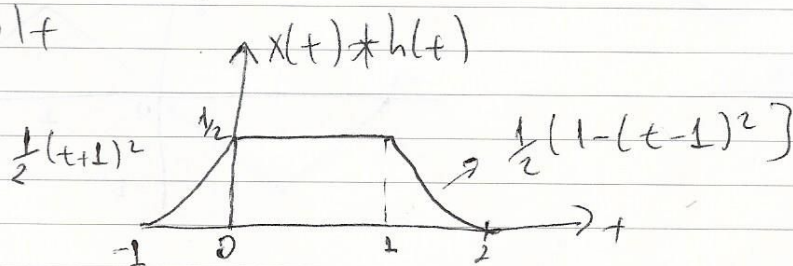
$\Rightarrow$



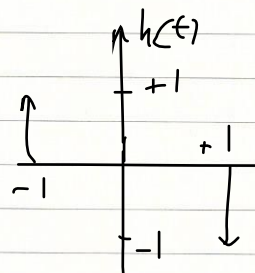
$\Rightarrow$



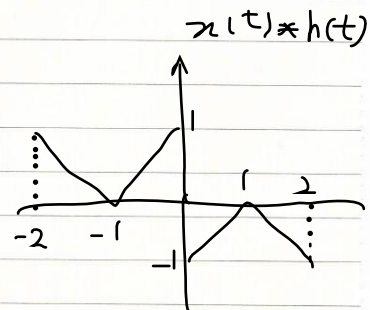
Result



*



=



9

$$2.5(c) \quad y(t) = \int_0^t [e^{-\tau} + 1] d\tau = [1 - e^{-t} + t] u(t)$$

$$\begin{aligned} e) \quad y(t) &= e^{-at} u(t) * (u(t) - e^{-at} u(t-b)) \\ &= e^{-at} u(t) * (u(t) - e^{-ab} e^{-a(t-b)} u(t-b)) \\ &= \frac{1}{a} (1 - e^{-at}) u(t) - (t-b) e^{-at} u(t-b) \end{aligned}$$

$$2.11. (b) \quad h(t) = e^{4t} u(-t)$$

(i) Noncausal. because $h(t) \neq 0$ for $t < 0$

(ii) Stable because $h(t)$ is absolutely integrable.

$$\int_{-\infty}^{\infty} e^{4t} u(-t) dt = \int_{-\infty}^0 e^{4t} dt = \left[\frac{e^{4t}}{4} \right]_{-\infty}^0 = \frac{1}{4} < \infty$$

$$(d) \quad h(t) = e^{-2|t|}$$

(i) Noncausal because $h(t) \neq 0$ for $t < 0$

(ii) Stable because $h(t)$ is absolutely integrable

$$\int_{-\infty}^{\infty} e^{-2|t|} dt = \int_{-\infty}^0 e^{2t} dt + \int_0^{\infty} e^{-2t} dt$$

$$= \frac{1}{2} + \frac{1}{2} = 1 < \infty$$

$$g) \quad h(t) = \delta(t) + e^{-3t} u(t)$$

i) causal $\rightarrow h(t) = 0$ for $t < 0$

$$ii) \text{ stable } \int_{-\infty}^{\infty} \delta(t) dt + \int_0^{\infty} e^{-3t} dt = 1 + \left(\frac{1}{3} \right) = \frac{4}{3} < \infty$$

(10)

(i) $h(t) = \delta'(t) + e^{-|2t|}$

(i) Noncausal because $h(t) \neq 0$ for $t < 0$.

(ii) Stable.

$$\begin{aligned} \int_{-\infty}^{\infty} \delta'(t) + e^{-|2t|} dt &= \int_{-\infty}^{\infty} \delta'(t) dt + \underbrace{\int_{-\infty}^0 e^{2t} dt + \int_0^{\infty} e^{-2t} dt}_h \\ &= \delta(t) + 1 \end{aligned}$$

we did in 2.11.12)

$$= \delta(t) + 1 < \infty$$

(11)

2.12.(e) $h(t) = e^{-t} u(t)$ is invertible and the
inverse system is $h_I(t) = \delta'(t) + \delta(t)$

$\Downarrow$
BIBO stable ∇

$$2.14. h(t) = h_5(t) * [h_1(t) + h_2(t) * h_3(t) + h_4(t)]$$

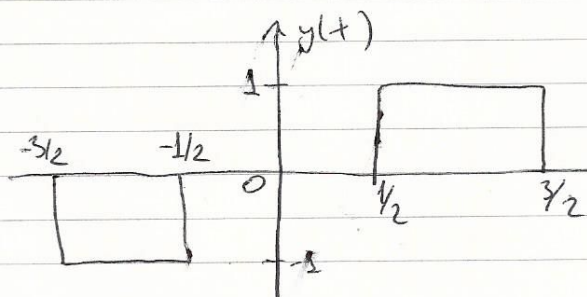
$$= e^{-3t} u(t) * [e^{-2t} u(t) + [e^{-t} - e^{-2t}] u(t) + \delta(t)]$$

$$= e^{-3t} u(t) * [e^{-t} u(t) + \delta(t)]$$

$$= \frac{1}{2} [e^{-t} - e^{-3t}] u(t) + e^{-3t} u(t)$$

$$= \frac{1}{2} [e^{-t} + e^{-3t}] u(t)$$

2.15.(c)



(d)

