

Homework 3 Solutions

Oppenheim Book

3.3. The given signal is

$$x(t) = 2 + \frac{1}{2} e^{j(2\pi/3)t} + \frac{1}{2} e^{-j(2\pi/3)t} - 2j e^{j(5\pi/3)t} + 2j e^{-j(5\pi/3)t}$$

$$x(t) = 2 + \frac{1}{2} e^{j2(2\pi/6)t} + \frac{1}{2} e^{-j2(2\pi/6)t} - 2j e^{j5(2\pi/6)t} + 2j e^{-j5(2\pi/6)t}$$

From this, we may conclude that the fundamental frequency of $x(t)$ is $2\pi/6 = \pi/3$. The non-zero Fourier series coefficients of $x(t)$ are:

$$a_0 = 2, \quad a_2 = a_{-2} = \frac{1}{2}, \quad a_5 = a_{-5}^* = -2j$$

3.21. Using the Fourier series synthesis equation:

(2)

$$x(t) = a_1 e^{j(2\pi/T)t} + a_{-1} e^{-j(2\pi/T)t} + a_5 e^{j5(2\pi/T)t} + a_{-5} e^{-j5(2\pi/T)t}$$

$$= j e^{j(2\pi/8)t} - j e^{-j(2\pi/8)t} + 2 e^{j5(2\pi/8)t} + 2 e^{-j5(2\pi/8)t}$$

$$= -2 \sin\left(\frac{\pi}{4}t\right) + 4 \cos\left(\frac{5\pi}{4}t\right)$$

$$= -2 \cos\left(\frac{\pi}{4}t - \frac{\pi}{2}\right) + 4 \cos\left(\frac{5\pi}{4}t\right) //$$

3.22) b-) $T=2$, $a_k = \frac{-j^k}{2(1+jk\pi)} [e - e^{-1}]$ for all k .

c-) $T=3$, $\omega_0 = 2\pi/3$, $a_0 = 1$ and

$$a_k = \frac{2e^{-j\pi k/3}}{\pi k} \sin(2\pi k/3) + \frac{e^{-j\pi k}}{\pi k} \sin(\pi k)$$

3.25-) a-) The non-zero FS coefficients of $x(t)$ are

$$a_1 = a_{-1} = 1/2$$

b-) The non-zero FS coefficients of $x(t)$ are

$$b_1 = b_{-1} = 1/2j$$

c-) Using the multiplication property, we know that (3)

$$z(t) = x(t)y(t) \xrightarrow{\text{FS}} c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

Therefore,

$$c_k = a_k * b_k = \frac{1}{4j} \delta[k-2] - \frac{1}{4j} \delta[k+2].$$

d-) We have

$$z(t) = \sin(4t) \cos(4t) = \frac{1}{2} \sin(8t)$$

Therefore, the non zero Fourier series coefficients of $z(t)$ are $c_2 = c_{-2} = (1/4j)$

3.26. a-) If $x(t)$ is real, then $x(t) = x^*(t)$. This implies that for $x(t)$ real $a_k = a_{-k}^*$. Since this is not true in this problem, $x(t)$ is not real.

b-) If $x(t)$ is even, then $x(t) = x(-t)$ and $a_k = a_{-k}$. Since this is true for this case, $x(t)$ is even.

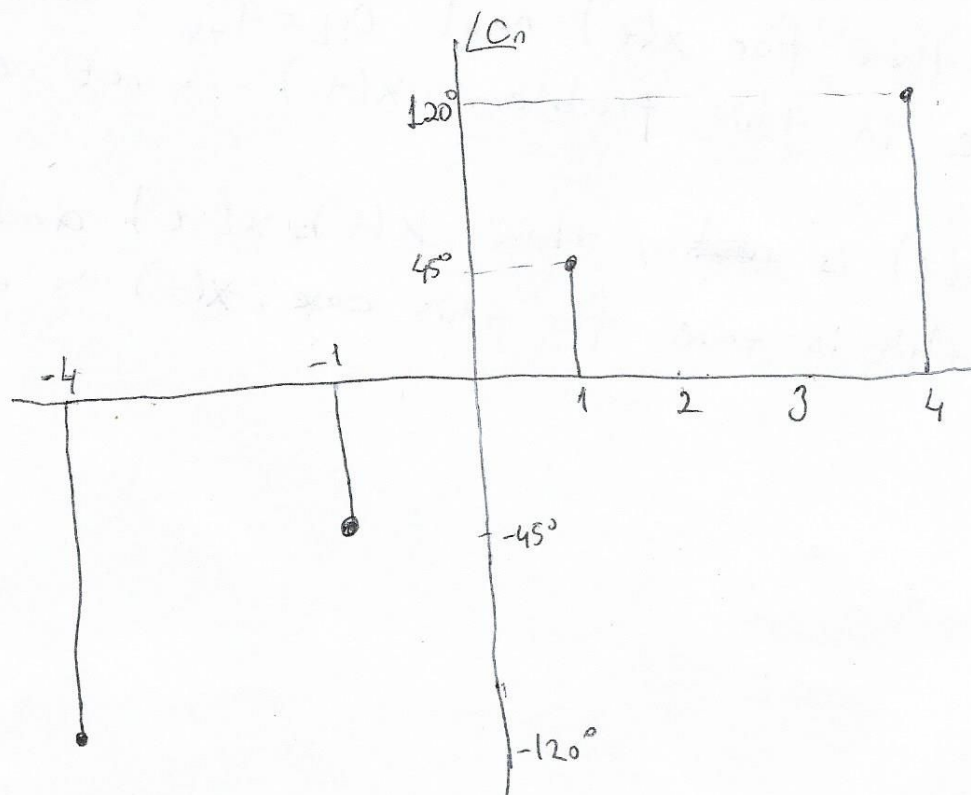
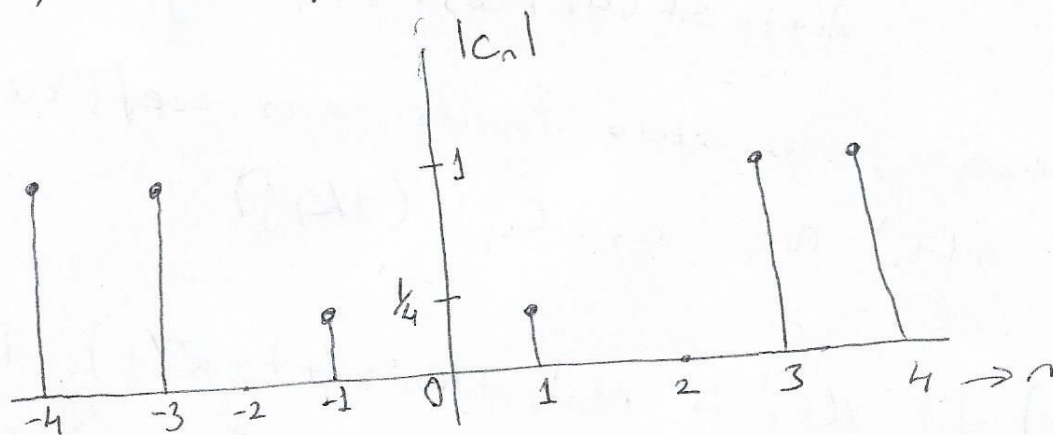
3.7-) (a) With $\omega_0 = 1$, we can write

$$x(t) = 2 + \frac{1}{4} \left[e^{j(\omega_0 t + 45^\circ)} + e^{-j(\omega_0 t + 45^\circ)} \right] + \left[e^{j3\omega_0 t} + e^{-j3\omega_0 t} \right] + j \left[e^{j(4\omega_0 t + 30^\circ)} - e^{-j(4\omega_0 t + 30^\circ)} \right]$$

So that

$$c_0 = 2, \quad c_1 = c_{-1}^* = \frac{1}{4} e^{j45^\circ}, \quad c_3 = c_{-3} = 1, \quad c_4 = c_{-4}^* = j e^{j30^\circ} = e^{j120^\circ}$$

(b)



3.10. $x(t) = \sin t$ $0 \leq t \leq \pi$ $x(t+\pi) = x(t)$ (5)

$$C_n = \frac{1}{T} \int_T x(t) \exp[-j2\pi nt] dt$$

$$= \frac{1}{\pi} \int_0^\pi \sin t \exp[-j2nt] dt = \frac{1}{\pi} \int_0^\pi \left[\frac{\exp[jt] - \exp[-jt]}{2j} \right] \exp[-j2nt] dt$$

$$= \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right)$$

3.18- (i) $C_n = \frac{1}{T} \int_T x(t) \exp\left[-j\frac{2\pi n t}{T}\right] dt$

$$= \int_{-1/2}^{1/2} f(t) \exp[-j2\pi n t] dt = 1$$

(ii) $x(t) = -f(t) + f(t-1)$ $x(t+2) = x(t)$

$$C_n = \frac{1}{T} \int_T x(t) \exp\left[-j\frac{2\pi n t}{T}\right] dt$$

$$= \frac{1}{2} \int_{-1/2}^{3/2} [-f(t) + f(t-1)] \exp[-jn\pi t] dt$$

$$= \frac{1}{2} [-1 + \exp[-jn\pi]]$$

3.22. [a] $z(t) = \frac{1}{T} \int_{\langle T \rangle} x(\tau) y(t-\tau) d\tau$

(6)

$$\begin{aligned} z(t+T) &= \frac{1}{T} \int_{\langle T \rangle} x(\tau) y(t+T-\tau) d\tau \\ &= \frac{1}{T} \int_{\langle T \rangle} x(\tau) y(t-\tau) d\tau \end{aligned}$$

$\therefore z(t)$ is periodic with period T .

3.35. $y_c(t) = \int_{-\infty}^{\infty} x(\tau) \cos n\omega_0 \tau h(t-\tau) d\tau$

$$= \frac{1}{T_1} \int_{t-T_1}^t x(\tau) \cos n\omega_0 \tau d\tau$$

$$y_s(t) = \int_{-\infty}^{\infty} x(\tau) \sin n\omega_0 \tau h(t-\tau) d\tau$$

$$= \frac{1}{T_1} \int_{t-T_1}^t x(\tau) \sin n\omega_0 \tau d\tau$$

Also $C_n = \frac{1}{T} \int_0^T x(\tau) \exp[-jn\omega_0 \tau] d\tau$

$$= \frac{1}{T} \int_0^T x(\tau) \cos n\omega_0 \tau d\tau - j \frac{1}{T} \int_0^T x(\tau) \sin n\omega_0 \tau d\tau$$

$$= y_c(t) - j y_s(t)$$

(7)

$$(a) T_1 = T$$

$$y_c(t) = \frac{1}{T} \int_{t-T}^t x(\tau) \cos n\omega_0 \tau d\tau$$

$$= \frac{1}{T} \int_0^T x(\tau) \cos n\omega_0 \tau d\tau = \operatorname{Re} \{C_n\}$$

$$y_s(t) = \frac{1}{T} \int_{t-T}^t x(\tau) \sin n\omega_0 \tau d\tau$$

$$= \frac{1}{T} \int_0^T x(\tau) \sin n\omega_0 \tau d\tau$$

$$= -\operatorname{Im} \{C_n\}$$

$$(b) T_1 = lT,$$

$$y_c(t) = \frac{1}{lT} \int_{t-lT}^t x(\tau) \cos n\omega_0 \tau d\tau$$

$$= \frac{l}{lT} \int_0^T x(\tau) \cos n\omega_0 \tau d\tau$$

$$= \frac{1}{T} \int_0^T x(\tau) \cos n\omega_0 \tau d\tau$$

$$= \operatorname{Re} \{C_n\}$$

$$y_s(t) = \frac{1}{lT} \int_{t-lT}^t x(\tau) \sin n\omega_0 \tau d\tau$$

(8)

$$= \frac{1}{lT} \int_0^T x(\tau) \sin n\omega_0 \tau d\tau$$

$$= \frac{1}{T} \int_0^T x(\tau) \sin n\omega_0 \tau d\tau$$

$$= -\text{Im} \{C_n\}$$

(c) $T_1 \gg T$ but $T_1 \neq lT$

Assume $T_1 = lT + fT$ $l = \text{integer}$, $0 < f < 1$ and $l \gg f$

$$y_c(t) = \frac{1}{lT + fT} \int_{t-T_1}^t x(\tau) \cos n\omega_0 \tau d\tau$$

$$= \frac{1}{lT + fT} \int_{t-T_1}^{(t-T_1)+lT} x(\tau) \cos n\omega_0 \tau d\tau$$

$$+ \frac{1}{lT + fT} \int_{(t-T_1)+lT}^{(t-T_1)+lT+fT} x(\tau) \cos n\omega_0 \tau d\tau$$

Since $lT \gg fT$

$$y_c(t) \approx \frac{1}{lT} \int_0^{fT} x(\tau) \cos n\omega_0 \tau d\tau$$

$$+ \frac{1}{lT} \int_0^{fT} x(\tau) \cos n\omega_0 \tau d\tau$$

$$\approx \frac{1}{T} \int_0^T x(\tau) \cos n\omega_0 \tau d\tau = \text{Re} \{C_n\}$$

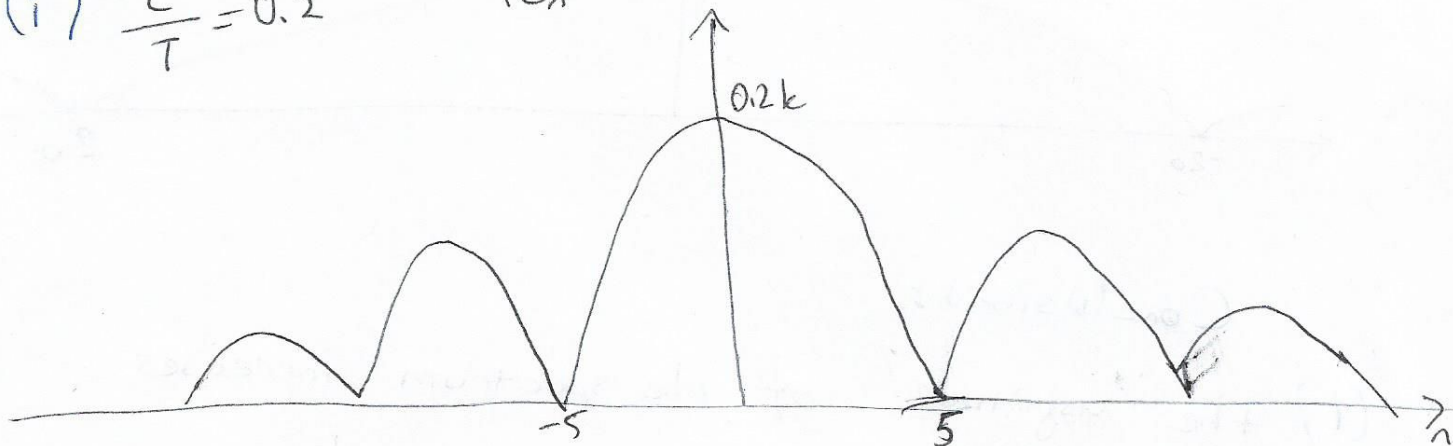
Similarly $y_s(t) \approx -\text{Im} \{C_n\}$

(9)

3.40. $C_n = \frac{k\tau}{T} \sin \tau (n\pi \tau/T)$ (from eq 3.3.17 in Soliman Book)

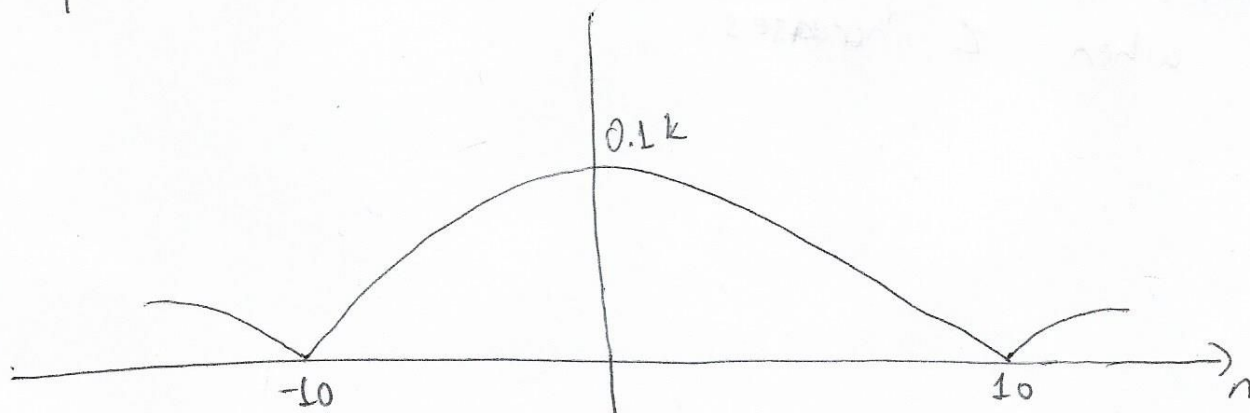
(i) $\frac{\tau}{T} = 0.2$

$|C_n| = 0.2k \sin \tau (0.2n\pi)$



(ii) $\frac{\tau}{T} = 0.1$

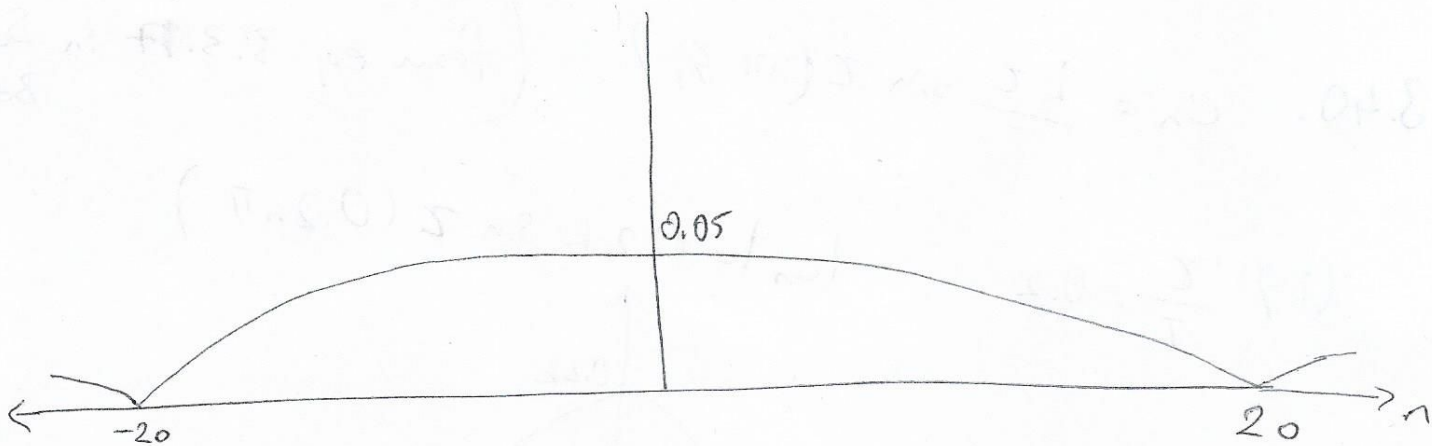
$|C_n| = 0.1k \sin \tau (0.1n\pi)$



(iii) $\frac{\tau}{T} = 0.05$

$$|c_n| = 0.05 K \sin \tau (0.05 n \pi)$$

(10)



Conclusions:

- (1) the magnitude of the spectrum increases proportional to τ ,
- (2) the frequency content of the signal is compressed when τ increases.